X = -2 => Y = (-2)2 = 4 X = -1 => Y = (-1)2 = 1 X = 1 => Y = 12 = 1 X = 2 => Y = 22 = 4

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X = | -2 | -1 | 1 | 2 |
| P(X) = |  |  |  |  |

II. 3. a.) Distributia JPD

Y = X2

🡺 Y = 4,1,1,4 => Y = 1, 4

P (X = -2, Y = 1) = 0 ((-2)2 != 1) P (X = -2, Y = 4) = ((-2)2 = 4) P (X = -1, Y = 1) = ((-1)2 = 1) P (X = -1, Y = 4) = 0 ((-1)2 != 4)

P (X = -1, Y = 1) = ((-1)2 = 1) P (X = -1, Y = 4) = 0 ((-1)2 != 4)

P (X = -2, Y = 1) = 0 ((-2)2 != 1) P (X = -2, Y = 4) = ((-2)2 = 4)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Y \ X | -2 | -1 | 1 | 2 |
| 1 | 0 |  |  | 0 |  |
| 4 |  | 0 | 0 |  |  |
|  |  |  |  |  | Adunare pe linii/coloane |

b.) Covariatia & Corelatia

Cov (X, Y) = E (X \* Y) – (E X) \* (E Y)

E X = (-2) \* + (-1) \* + 1 \* + 2 \* = E Y = 1 \* + 4 \* =

X \* Y = X \* X2 = X3 => E (X \* Y) = E (X3) = (-8) \* + (-1) \* + 1 \* + 8 \*

Cov (X, Y) = - \* =

Corelatia = 0 => X si Y sunt independente

ρ (X, Y) = =

Var (X) = E (X2) – (E X)2 = (-4) \* + (-1) \* + 1 \* + 4 \* - =

Var (Y) = E (Y2) – (E Y)2 = 1 \* + 16 \* - =

ρ (X, Y) =

Var (X) = Dispersia lui X

II. 6. a.) Repartitia comuna (JPD)

X = nr de bile albe

Y = nr de bile cu numarul 2

1 by 1, 2 balls (MAXIMUM)

U : 2w1 , 2w2 , 2b1 , 1b2

P (X=0, Y=0) = P (2b1) = = P (X=0, Y=1) = P (1b2) = = = P (X=0, Y=2) = P (2b2) = (Sunt doar 1b2) P (X=1, Y=0) = P (1w1, 1b1) = = P (X=1, Y=1) = P (1w2, 1b1) + P (1w1, 1b2) = + = P (X=1, Y=2) = P (1w2, 1b2) = = P (X=2, Y=0) = P (2w1) = = P (X=2, Y=1) = P (1w1+1w2) = = P (X=2, Y=2) = P (2w2) = =

|  |  |  |  |
| --- | --- | --- | --- |
| Y \ X | 0 | 1 | 2 |
| 0 |  |  |  |  |
| 1 |  |  |  |  |
| 2 | 0 |  |  |  |
|  |  |  |  | 1 |

b.) Verificarea daca variabilele sunt independente

P (X = Xi , Y = Yi) = P (X = Xi) \* P (Y = Yi­), pentru orice i si j => X si Y independent

P (X = 0, Y = 0) = =

P (X=0) \* P (Y=0) = \* =

}

=> P (X = 0, Y = 0) != P (X = 0) \* P (Y = 0) => X si Y sunt independente

II. 9.

A and B, three trows

B = 0.4t 0.6 h

X = nr of tails from A

Y = nr of tails from B

a.) Verify if X and Y are independent

Yes, they are physically independent due to the fact that X depends only on A and Y depends only on Y.

b.) Repartitia comuna (JPD)

P (X=0) = P (h,h,h) =

P (X=1) = P (t,h,h; h,t,h; h,h,t) =

P (X=2) = P (t,t,h; t,h,t; h,t,t) =

P (X=3) = P (t,t,t) =

(X ~ B (3, ) -- 3 with a 50% chance for each

Y ~ B (3, ) => P (Y=0) = ()3 (3t)

=> P (Y=1) = \* ()2 \* (2t 1h)

=> P (Y=2) = ()2 \*()2 \* (1t 2h)

=> P (Y=3) = ()3 (3h)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Y \ X | 0 | 1 | 2 | 3 |
| 0 |  |  |  |  |  |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
|  |  |  |  |  |  |

c.) P (X = Y) = =

P (X > Y) =

P (X + Y > = 4) =

III 2.)

Markov : X >= 0 , P (X >= t) <= , t > 0

M : P (X >= 2) <= = (t=2)

|X – 1| >= 2 🡪 => => X >= 3 (X >= 0) 🡺 P (|X-1| >= 2) = P (X>=3) <= =

P (X <= -3) = 0 , because X >= 0

III 2.)

Chebyshe : P (|X – E X | >= t ) <= , t > 0

2-1 (t-EX)

C :

P (X >= 2) = P (X-1 >= 1) <= P (|X-1| >= 1)

X-1 >= 1 => |X-1| >= 1 🡪{X-1 >= 1} apartine {|X-1| >= 1}

P (|X-1| >= 2) <= = 1

III 6.) 300 coin tosses, with the prob of head is 3/10, what is the prob to get head at least 100 times

X = nr of heads X ~ B ( 300, )

M : P (X >= 100) <= = = EX = 90 (300\*3/10) ---(n\*p) Var X : (n\*p\*(1-p))

C : P (X >= 100) = P (X - 90 >= 10) <= P (|X-90| >= 10) <= =

III 7.) n coin tosses, with the prob of 2/10, what is the prob to get head (at least) 50% of times

X = nr of heads X ~ B ( n, ) EX = n\*1/5 Var (X) = n\*1/5 \* (1- 1/5)

M : P (X >= ) <= = =

C : P (X >= ) = P (X - >= - ) <= P (|X - | >= ) <= =

III 8.) A = prob 1/4 for t, B = prob of 4/5, A and B are flipped 25 times, prob of 2t at least 10

X = success = 2t P (success) = 0.25 \* 0.8 = 0.2

M : P (X >=10) <= = =

C : P (X >=10) = P (X - 5 >= 5) <= P (|X - 5| >= 5) <= =